



# Reproducible and Accurate Matrix Multiplication for High-Performance Computing

Caroline Collange, David Defour, Stef Graillat, Roman Iakymchuk

## ► To cite this version:

Caroline Collange, David Defour, Stef Graillat, Roman Iakymchuk. Reproducible and Accurate Matrix Multiplication for High-Performance Computing. SCAN: Scientific Computing, Computer Arithmetic and Validated Numerics, Sep 2014, Wuerzburg, Germany. pp.42-43. hal-01215627

**HAL Id: hal-01215627**

**<https://hal.science/hal-01215627>**

Submitted on 23 Nov 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Reproducible and Accurate Matrix Multiplication for High-Performance Computing

Sylvain Collange, David Defour, Stef Graillat, and  
Roman Iakymchuk

INRIA – Centre de recherche Rennes – Bretagne Atlantique  
Campus de Beaulieu, F-35042 Rennes Cedex, France

`sylvain.collange@inria.fr`

DALI-LIRMM, Université de Perpignan

52 avenue Paul Alduy, F-66860 Perpignan, France

`david.defour@univ-perp.fr`

Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6

F-75005 Paris, France

CNRS, UMR 7606, LIP6, F-75005 Paris, France

Sorbonne Universités, UPMC Univ Paris 06, ICS

F-75005 Paris, France

`{stef.graillat, roman.iakymchuk}@lip6.fr`

**Keywords:** Matrix multiplication, reproducibility, accuracy, long accumulator, multi-precision, multi- and many-core architectures.

The increasing power of current computers enables one to solve more and more complex problems. This, therefore, requires to perform a high number of floating-point operations, each one leading to a round-off error. Because of round-off error propagation, some problems must be solved with a longer floating-point format.

As Exascale computing ( $10^{18}$  operations per second) is likely to be reached within a decade, getting accurate results in floating-point arithmetic on such computers will be a challenge. However, another challenge will be the reproducibility of the results – meaning getting a bitwise identical floating-point result from multiple runs of the same code – due to non-associativity of floating-point operations and dynamic scheduling on parallel computers.

Reproducibility is becoming so important that Intel proposed a “Conditional Numerical Reproducibility” (CNR) in its MKL (Math Kernel Library). However, CNR is slow and does not give any guarantee concerning the accuracy of the result. Recently, Demmel and Nguyen [1] proposed an algorithm for reproducible summation. Even though their algorithm is fast, no information is given on the accuracy.

More recently, we introduced [2] an approach to compute deterministic sums of floating-point numbers efficiently and with the best possible accuracy. Our multi-level algorithm consists of two main stages: filtering that relies upon fast vectorized floating-point expansions; accumulation which is based on superaccumulators in a high-radix carry-save representation. We presented implementations on recent Intel desktop and server processors, on Intel Xeon Phi accelerator, and on both AMD and NVIDIA GPUs. We showed that the numerical reproducibility and bit-perfect accuracy can be achieved at no additional cost for large sums that have dynamic ranges of up to 90 orders of magnitude by leveraging arithmetic units that are left underused by standard reduction algorithms.

In this talk, we will present a reproducible and accurate (rounding to the nearest) algorithm for the product of two floating-point matrices in parallel environments like GPU and Xeon Phi. This algorithm is based on the DGEMM implementation. We will show that the performance of our algorithm is comparable with the classic DGEMM.

## References:

- [1] J. DEMMEL, H. D. NGUYEN, Fast Reproducible Floating-Point Summation, *Proceeding of the 21st IEEE Symposium on Computer Arithmetic*, Austin, Texas, USA (2013), pp. 163-172.
- [2] S. COLLANGE, D. DEFOUR, S. GRAILLAT, R. IAKYMCHUK, Full-Speed Deterministic Bit-Accurate Parallel Floating-Point Summation on Multi- and Many-Core Architectures, *Research Report*. HAL ID: hal-00949355. February 2014.